



## Course and Examination Fact Sheet: Spring Semester 2021

### 8,302: Advanced Mathematics and Statistics

ECTS credits: 4

#### Overview examination/s

(binding regulations see below)

Decentral - Oral examination (individual) (100%)

Examination time: term time

#### Attached courses

Timetable -- Language -- Lecturer

[8,302,1.00 Advanced Mathematics and Statistics](#) -- Englisch -- [Aurino Francesco](#), [De Giorgi Enrico Giovanni](#)

#### Course information

#### Course prerequisites

##### Part Advanced Mathematics:

Mandatory MiQEF course Mathematics from the previous semester.

##### Part Advanced Statistics:

Mandatory MiQEF course Statistics from the previous Semester.

#### Learning objectives

##### Part Advanced Mathematics:

Students understand and apply advanced tools in mathematical finance to questions in Finance, e.g., asset pricing and portfolio selection. Students master the concepts in a way that they can read and understand research papers in finance applying advanced mathematical finance tools and results.

##### Part Advanced Statistics:

The course will prepare students for further studies in quantitative methods with applications in economics and finance and to produce rigorous research in these fields.

Students will learn how to apply theoretical statistical results to estimate key unknown quantities of the data generating distribution and to critically judge the goodness of a given estimator and, in case, how to correct for possible deficiencies.

#### Course content

##### Part Advanced Mathematics:

Course in Stochastic Calculus with applications to Finance.

We introduce stochastic calculus and some of its applications in Finance. We first define basic concepts in probability theory, as filtered probability spaces, conditional expectations and martingales. We then define the stochastic integral for simple processes as well as for general processes. We present three crucial results in stochastic calculus - the Ito Lemma, the Girsanov Theorem and the Martingale Representation Theorem, - and discuss their relevance for Finance.



The lectures combine theoretical parts with exercises (four exercise series will be distributed and discussed during the sessions).

The lectures will equip students with fundamental theoretical results in mathematical finance needed to study asset pricing and portfolio selection. Students will learn the mathematical setup developed in mathematical finance in order to price financial instruments and derive dynamic investment strategies.

## Part Advanced Statistics:

Advanced course in the statistical theory.

During the course, we will consider two/three main statistical topics that are central for many practical applications in economics and finance, namely: the main asymptotical results for sequences of sums and maxima (extreme value theory), a detailed discussion of the main properties of estimators, and how to select a best estimator (minimum variance unbiased estimation).

## Course structure

### Part Advanced Mathematics:

#### 1. Probability Spaces and Stochastic Processes

1.1. Introduction 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration 1.3. Conditional Expectation 1.4. Martingales 1.5. Radon-Nikodým Theorem 1.6. Brownian Motion

2. **Stochastic Integral** 2.1. Motivation 2.2. Simple Integrand 2.3. General Integrand

3. **Itô Calculus** 3.1. Itô Processes 3.2. Itô Lemma 3.3. Stochastic Differential Equations

4. **Girsanov Theorem and Martingale Representation Theorem** 4.1. Girsanov Theorem 4.2. Martingale Representation Theorem 4.3. Applications in Finance

### Part Advanced Statistics:

#### 1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
- Fluctuations of Maxima
  - Limit probabilities for maxima
  - Weak convergence of maxima under affine transformations
  - The extreme value distributions

#### 2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

## Course literature

### Part Advances Mathematics:

- De Giorgi, Enrico (2021): Advanced Mathematics and Statistics - Part Advanced Mathematics, Lecture Notes, HSG.
- Shreve, Steven E. (2004): *Stochastic Calculus for Finance*, Vol. II, Springer. (suggested)

Additional references to the specific topics will be provided during the course.

### Part Advanced Statistics:



- F. Audrino, Lecture Notes.
- E.J. Dudewicz and S.N. Mishra, *Modern mathematical statistics*, Wiley, New York, 1988. (suggested)

Additional references to the specific topics will be provided during the course.

## Additional course information

In the case of the President's Board having to implement new directives due to the SARS-CoV-2 pandemic in the Spring Semester 2021, the course information listed above will be changed as follows:

- In case face-to-face teaching for *all* students is not possible (full lockdown, impossibility to guarantee social distancing, many students are constraint in joining classes in St.Gallen) the course is conducted online via the platform Zoom;
- The recordings of the course are available with the Zoom cloud (access via learning.unisg.ch);
- The lecturer informs via StudyNet (learning.unisg.ch) and e-mail on the changed implementation modalities of the course.

The examination information listed below would be changed as follows:

- The oral examinations are conducted online via the platform Zoom;
- The examination modality and further information are communicated via StudyNet and e-mail.

## Examination information

### Examination sub part/s

#### 1. Examination sub part (1/1)

##### Examination time and form

Decentral - Oral examination (individual) (100%)

Examination time: term time

##### Remark

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##### Examination-aid rule

Extended Closed Book

The use of aids is limited; any additional aids permitted are exhaustively listed under "Supplementary aids". Basically, the following is applicable:

- At such examinations, all the pocket calculators of the Texas Instruments TI-30 series and mono- or bilingual dictionaries (no subject-specific dictionaries) without hand-written notes are admissible. Any other pocket calculator models and any electronic dictionaries are inadmissible.
- In addition, any type of communication, as well as any electronic devices that can be programmed and are capable of communication such as notebooks, tablets, mobile telephones and others, are inadmissible.
- Students are themselves responsible for the procurement of examination aids.

##### Supplementary aids

None.

##### Examination languages

Question language: English

Answer language: English

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## Examination content



## Part Advanced Mathematics:

**1. Probability Spaces and Stochastic Processes** 1.1. Introduction 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration 1.3. Conditional Expectation 1.4. Martingales 1.5. Radon-Nikodým Theorem 1.6. Brownian Motion

**2. Stochastic Integral** 2.1. Motivation 2.2. Simple Integrands 2.3. General Integrands

**3. Itô Calculus** 3.1. Itô Processes 3.2. Itô Lemma 3.3. Stochastic Differential Equations

**4. Girsanov Theorem and Martingale Representation Theorem** 4.1. Girsanov Theorem 4.2. Martingale Representation Theorem 4.3. Applications in Finance

## Part Advanced Statistics:

### 1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
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  - Weak convergence of maxima under affine transformations
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### 2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

## Examination relevant literature

### Part Advanced Mathematics:

De Giorgi, Enrico (2021): Advanced Mathematics and Statistics - Part Advanced Mathematics, Lecture Notes and Exercises, HSG. The lecture notes and exercisers are available on StudyNet.

### Part Advanced Statistics:

F. Audrino, Lecture Notes, available on StudyNet at the beginning of the course.



### Please note

Please note that only this fact sheet and the examination schedule published at the time of bidding are binding and takes precedence over other information, such as information on StudyNet (Canvas), on lecturers' websites and information in lectures etc.

Any references and links to third-party content within the fact sheet are only of a supplementary, informative nature and lie outside the area of responsibility of the University of St.Gallen.

Documents and materials are only relevant for central examinations if they are available by the end of the lecture period (CW21) at the latest. In the case of centrally organised mid-term examinations, the documents and materials up to CW 12 are relevant for testing.

Binding nature of the fact sheets:

- Course information as well as examination date (organised centrally/decentrally) and form of examination: from bidding start in CW 04 (Thursday, 28 January 2021);
- Examination information (regulations on aids, examination contents, examination literature) for decentralised examinations: in CW 12 (Monday, 22 March 2021);
- Examination information (regulations on aids, examination contents, examination literature) for centrally organised mid-term examinations: in CW 12 (Monday, 22 March 2021);
- Examination information (regulations on aids, examination contents, examination literature) for centrally organised examinations: two weeks before the end of the registration period in CW 14 (Thursday, 8 April 2021).