

Course and Examination Fact Sheet: Spring Semester 2020

8,302: Advanced Mathematics and Statistics

ECTS credits: 4

Overview examination/s (binding regulations see below) Decentral - Oral examination (individual) (100%, 20 mins.)

Attached courses

Timetable -- Language -- Lecturer 8,302,1.00 Advanced Mathematics and Statistics -- Englisch -- Audrino Francesco, De Giorgi Enrico

Course information

Course prerequisites

Part Advanced Mathematics:

Mandatory MiQEF course Mathematics from the previous semester.

Part Advanced Statistics:

Mandatory MiQEF course Statistics from the previous Semester.

Learning objectives

Part Advanced Mathematics:

Students understand and apply advanced tools in mathematical finance to questions in Finance, e.g., asset pricing and portfolio selection. Students master the concepts in a way that they can read and understand research papers in finance applying advanced mathematical finance tools and results.

Part Advanced Statistics:

The course will prepare students for further studies in quantitative methods with applications in economics and finance and to produce rigorous research in these fields.

Students will learn how to apply theoretical statistical results to estimate key unkwnown quantities of the data generating distribution and to critically judge the goodness of a given estimator and, in case, how to correct for possible deficiencies.

Course content

Part Advanced Mathematics:

Course in Stochastic Calculus with applications to Finance.

We introduces stochastic calculus and some of its applications in Finance. We first define basic concepts in probability theory, as filtered probability spaces, conditional expectations and martingales. We then define the stochastic integral for simple processes as well as for general processes. We present three crucial results in stochastic calculus - the Ito Lemma, the Girsanov Theorem and the Martingale Representation Theorem, - and discuss their relevance for Finance.

The lectures combine theoretical parts with exercises (four exercise series will be distributed and discussed during the sessions).



The lectures will equip students with fundamental theoretical results in mathematical finance needed to study asset pricing and portfolio selection. Students will learn the mathematical setup developed in mathematical finance in order to price financial instruments and derive dynamic investment strategies.

Part Advanced Statistics:

Advanced course in the statistical theory.

During the course, we will consider two/three main statistical topics that are central for many practical applications in economics and finance, namely: the main asymptotical results for sequences of sums and maxima (extreme value theory), a detailed discussion of the main properties of estimators, and how to select a best estimator (minimum variance unbiased estimation).

Course structure

Part Advanced Mathematics:

1. Probability Spaces and Stochastic Processes

1.1. Introduction 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration 1.3. Conditional Expectation 1.4. Martingales 1.5. Radon-Nikodým Theorem 1.6. Brownian Motion

2. Stochastic Integral 2.1. Motivation 2.2. Simple Integrands 2.3. General Integrands

3. Itô Calculus 3.1. Itô Processes 3.2. Itô Lemma 3.3. Stochastic Differential Equations

4. Girsanov Theorem and Martingale Representation Theorem 4.1. Girsanov Theorem **4**.2. Martingale Representation Theorem **4**.3. Applications in Finance

Part Advanced Statistics:

1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
- Fluctuations of Maxima
 - Limit probabilities for maxima
 - Weak convergence of maxima under affine transformations
 - The extreme value distributions

2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

Course literature

Part Advances Mathematics:

• De Giorgi, Enrico (2020): Advanced Mathematics and Statistics - Part Advanced Mathematics, Lecture Notes, HSG. Shreve, Steven E. (2004): *Stochastic Calculus for Finance*, Vol. II, Springer. (suggested)

Part Advanced Statistics:

F. Audrino, Leture Notes.

E.J. Dudewicz and S.N. Mishra, Modern mathematical statistics, Wiley, New York, 1988. (suggested)



Additional course information

Examination information

Examination sub part/s

1. Examination sub part (1/1)

Examination time and form

Decentral - Oral examination (individual) (100%, 20 mins.)

Remark

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Examination-aid rule

Extended Closed Book

The use of aids is limited; any additional aids permitted are exhaustively listed under "Supplementary aids". Basically, the following is applicable:

- At such examinations, all the pocket calculators of the Texas Instruments TI-30 series and mono- or bilingual dictionaries (no subject-specific dictionaries) without hand-written notes are admissible. Any other pocket calculator models and any electronic dictionaries are inadmissible.
- In addition, any type of communication, as well as any electronic devices that can be programmed and are capable of communication such as notebooks, tablets, PDAs, mobile telephones and others, are inadmissible.
- Students are themselves responsible for the procurement of examination aids.

Supplementary aids None.

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Examination languages Question language: English Answer language: English

Examination content

Part Advanced Mathematics:

1. Probability Spaces and Stochastic Processes 1.1. Introduction 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration 1.3. Conditional Expectation 1.4. Martingales 1.5. Radon-Nikodým Theorem 1.6. Brownian Motion

2. Stochastic Integral 2.1. Motivation 2.2. Simple Integrands 2.3. General Integrands

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Part Advanced Statistics:

1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
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• The extreme value distributions

2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

Examination relevant literature

Part Advanced Mathematics:

De Giorgi, Enrico (2020): Advanced Mathematics and Statistics - Part Advanced Mathematics, Lecture Notes and Exercises, HSG. The lecture notes and exercisers are available on StudyNet.

Part Advanced Statistics:

F. Audrino, Lecture Notes, available on StudyNet.

Please note

Please note that this fact sheet alone is binding and has priority over any other information such as StudyNet (Canvas), personal databases or faculty members' websites and information provided in their lectures, etc.

Any possible references and links within the fact sheet to information provided by third parties are merely supplementary and informative in nature and are outside the University of St.Gallen's scope of responsibility and guarantee.

Documents and materials that have been submitted no later than the end of term time (CW21) are relevant to central examinations.

Binding nature of the fact sheet:

- Information about courses and examination time (central/decentral) and examination typestarting from the beginning of the bidding on 23 January 2020
- Information about examinations (examination aid regulations, examination content, examination-relevant literature) for decentral examinations after the 4th semester week on 16 March 2020
- Information about examinations (examination aid regulations, examination content, examination-relevant literature) for central examinations as from the starting date for examination registration on 6 April 2020

Please consult the fact sheet again after these deadlines have expired.