



## Course and Examination Fact Sheet: Spring Semester 2015

### 8,302: Advanced Mathematics and Statistics

ECTS credits: 4

#### Overview examination/s

(binding regulations see below)

Decentral - Oral examination (individual) (100%, 20 mins.)

#### Attached courses

Timetable -- Language -- Lecturer

[8,302,1.00 Advanced Mathematics and Statistics](#) -- English -- [Audrino Francesco](#), [De Giorgi Enrico](#)

#### Course information

#### Course prerequisites

##### Part Advanced Mathematics:

Mandatory MiQEF course Mathematics from the previous semester.

##### Part Advanced Statistics:

Mandatory MiQEF course Statistics from the previous semester.

#### Course content

##### Part Advanced Mathematics:

We introduce stochastic calculus and some of its applications in Finance. We first define basic concepts in probability theory, as filtered probability spaces, conditional expectations and martingales. We then define the stochastic integral for simple processes first and for general processes then. We present three crucial results in stochastic calculus - the Ito Lemma, the Girsanov Theorem and the Martingale Representation Theorem, - and discuss their relevance for Finance.

The lectures combine theoretical parts with exercises (four exercise series will be distributed and discussed during the sessions).

##### Part Advanced Statistics:

During the course, we will consider two/three main statistical topics that are central for many practical applications in economics and finance, namely: extreme value theory, minimum variance unbiased estimation, and non-parametric inference.

#### Course structure

##### Part Advanced Mathematics:

##### 1. Probability Spaces and Stochastic Processes

- 1.1. Introduction
- 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration
- 1.3. Conditional Expectation
- 1.4. Martingales
- 1.5. Radon-Nikodým Theorem
- 1.6. Brownian Motion



## 2. Stochastic Integral

- 2.1. Motivation
- 2.2. Simple Integrands
- 2.3. General Integrands

## 3. Itô Calculus

- 3.1. Itô Processes
- 3.2. Itô Lemma
- 3.3. Stochastic Differential Equations

## 4. Girsanov Theorem and Martingale Representation Theorem

- 4.1. Girsanov Theorem
- 4.2. Martingale Representation Theorem
- 4.3. Applications in Finance

### Part Advanced Statistics:

#### 1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
- Fluctuations of Maxima
  - Limit probabilities for maxima
- Weak convergence of maxima under affine transformations
- The extreme value distributions

#### 2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

#### 3. An introduction to nonparametric inference

- Nonparametric confidence intervals
- The Kolmogorov-Smirnov test
- The Lilliefors test for normality

## Course literature

### Part Advances Mathematics:

De Giorgi, Enrico (2015): Advanced Mathematics and Statistics – Part Advanced Mathematics, Lecture Notes, HSG.  
Shreve, Steven E. (2004): Stochastic Calculus for Finance, Vol. II, Springer. (suggested)

### Part Advanced Statistics:

F. Audrino, Lecture Notes.

E.J. Dudewicz and S.N. Mishra, *Modern mathematical statistics*, Wiley, New York, 1988. (suggested)

## Additional course information



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## Examination information

### Examination sub part/s

#### 1. Examination sub part (1/1)

##### Examination time and form

Decentral - Oral examination (individual) (100%, 20 mins.)

##### Remark

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##### Examination-aid rule

Extended Closed Book

The use of aids is limited; any additional aids permitted are **exhaustively** listed under "Supplementary aids". Basically, the following is applicable:

- At such examinations, all the pocket calculators of the Texas Instruments **TI-30** series and bilingual dictionaries without hand-written notes are admissible. Any other pocket calculator models and any electronic dictionaries are inadmissible.
- In addition, any type of communication, as well as any electronic devices that can be programmed and are capable of communication such as notebooks, tablets, PDAs, mobile telephones and others, are inadmissible.
- Students are themselves responsible for the procurement of examination aids.

##### Supplementary aids

None.

##### Examination languages

Question language: English

Answer language: English

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## Examination content

### Part Advanced Mathematics:

#### 1. Probability Spaces and Stochastic Processes

- 1.1. Introduction
- 1.2. Filtered Probability Space: Probability Measure, sigma-Algebra, Filtration
- 1.3. Conditional Expectation
- 1.4. Martingales
- 1.5. Radon-Nikodým Theorem
- 1.6. Brownian Motion

#### 2. Stochastic Integral

- 2.1. Motivation
- 2.2. Simple Integrands
- 2.3. General Integrands

#### 3. Itô Calculus

- 3.1. Itô Processes
- 3.2. Itô Lemma
- 3.3. Stochastic Differential Equations



## 4. Girsanov Theorem and Martingale Representation Theorem

4.1. Girsanov Theorem

4.2. Martingale Representation Theorem

4.3. Applications in Finance

### Part Advanced Statistics:

#### 1. Extreme value distributions

- Fluctuation of Sums: the law of large numbers and the central limit theorem
- Fluctuations of Maxima
- Limit probabilities for maxima
- Weak convergence of maxima under affine transformations
- The extreme value distributions

#### 2. Minimum-variance unbiased estimation

- "Goodness" properties of estimators
- Cramer-Rao inequality and efficiency of estimation
- Sufficiency and completeness
- Best unbiased estimators

## Examination relevant literature

### Part Advanced Mathematics:

De Giorgi, Enrico (2015): Advanced Mathematics and Statistics – Part Advanced Mathematics, Lecture Notes, HSG. The lecture notes will be available on StudyNet and will be published entirely until March 16th, 2015.

### Part Advanced Statistics:

F. Audrino, Lecture Notes.

### Please note

We would like to point out to you that this fact sheet has absolute priority over other information such as StudyNet, faculty members' personal databases, information provided in lectures, etc.

When will the fact sheets become binding?

- Information about courses and examination time (central/decentral and grading form): from the start of the bidding process on 22 January 2015
- Information about decentral examinations (examination-aid rule, examination content, examination relevant literature): after the 4th semester week on 16 March 2015
- Information about central examinations (examination-aid rule, examination content, examination relevant literature): from the start of the enrolment period for the examinations on 6 April 2015

Please look at the fact sheet once more after these deadlines have expired.